



Spectral action and gravitational effects at the Planck scale

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ARTICLE INFO

Article history:

Received 17 November 2013

Received in revised form 7 January 2014

Accepted 12 January 2014

Available online 15 January 2014

Editor: L. Alvarez-Gaumé

ABSTRACT

We discuss the possibility to extend the spectral action up to energy close to the Planck scale, taking also into account the gravitational effects given by graviton exchange. Including this contribution in the theory, the coupling constant unification is not compromised but is shifted to the Planck scale rendering all gauge couplings asymptotically free. In the scheme of noncommutative geometry, the gravitational effects change the main standard model coupling constants, leading to a restriction of the free parameters of the theory compatible with the Higgs and top mass prediction. We also discuss consequences for the neutrino mass and the see-saw mechanism.

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1. Introduction

Noncommutative geometry [1–4] allows to handle a large variety of geometrical frameworks from a totally algebraic point of view. In particular it is very useful in the derivation of models in high energy physics, such as the Yang–Mills gauge theories [5–9]. In the current state the noncommutative geometry structure of gauge theories is understood to be an “almost commutative” geometry, i.e. the product of continuous geometry, representing space–time, times an internal algebra of finite dimensional matrices. In this geometric framework the spectral action principle [10] enables the retrieval of the full standard model of high energy physics, including the Higgs field: the standard model is put on the same footing as geometrical general relativity, making it possible the unification with gravity. In fact, the application of noncommutative geometry to gauge theories of strong and electroweak forces is a very original way to fully geometrize the interaction of elementary particles. Furthermore, it has been shown [11] that it is possible to extend the standard model by including an additional singlet scalar field that stabilizes the running coupling constants of the Higgs field. This singlet scalar field is closely related to the right-handed Majorana neutrinos, conferring them mass, and leading to the prediction of the seesaw mechanism which explains the large difference between the masses of neutrinos and those of the other fermions. A recent model [12] shows the possibility of a further extension, going one step higher in the construction of the noncommutative manifold, in a sort of noncommutative geometry grand unification: here it is pointed out that there could be a “next level” in noncommutative geometry, intertwined with the

Riemannian and spin structure of space–time, where the singlet-scalar field arises. Accordingly, it naturally appears at high scale, near to the Planck scale.

A possible framework for describing interactions at energies and momenta below the Planck scale is given in [13,14]. In this Letter we check the possibility to extend the unification scale up to the Planck scale $M_P \equiv \sqrt{\hbar c/G_N} \simeq 10^{19}$ GeV, including not negligible gravitational effects. For a theory dealing with the unification of gauge theory and gravity, a more natural scale is the Planck scale. The usual strategy is to use the spectral action as an effective action at a fixed scale, of the order of the unification scale, and to impose the additional relations between the independent parameters of the standard model. Then, using the renormalization group (RG) equations, one can let these parameters run to their value at low scales and evaluate the Higgs, the top and neutrino masses. The question here is: what is the predictive power of this extended model with exchange of gravitons at the Planck scale? We want to see how the gravitational effects change the main running coupling constants and if they lead to a restriction on the free parameters of the theory still compatible with the Higgs, top and neutrino mass predictions.

In [15] Marcolli and Estrada carried out a similar analysis within the asymptotic safety scenario with Gaussian matter fixed point; differently from this Letter, they have not considered the effect of the scalar field σ introduced in [11], which is necessary in order to reproduce the seesaw mechanism and to have the Higgs mass with its correct value.

This Letter is organized as follows. In Section 2, some ingredients and the main results of the spectral action principle are shown: the derivation of the full standard model bosonic action plus the singlet scalar field and gravity. In Section 3, the

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gravitational contributions to the three gauge couplings, not negligible at the Planck scale, are presented. In Section 4, it is shown how the gravitational effects change the RG equations of the Yukawa and self-interaction Higgs couplings leading to a restriction of the free parameters of the theory compatible with the Higgs and top mass. The final section contains conclusions and some comments.

2. The spectral action

We recall the main features of the spectral action, referring to the original works [1,10] for the full treatment. Those familiar with this calculation can skip to the next section.

The basic ingredients of noncommutative geometry are: an algebra \mathcal{A} , which involves the topology of space-time and its non-commutative generalization, a Hilbert space \mathcal{H} on which the algebra acts, containing the fermionic degrees of freedom, and a generalized Dirac operator D which encodes the metric structure of the space. These three objects form the so called spectral triple. The triple is said to be even if there is an operator Γ on \mathcal{H} such that $\Gamma = \Gamma^*$, $\Gamma^2 = 1$ and

$$\Gamma D + D \Gamma = 0; \quad \Gamma a - a \Gamma = 0, \quad \forall a \in \mathcal{A}. \quad (2.1)$$

A spectral triple, enlarged with an anti-unitary operator J on \mathcal{H} that obeys to: 1) $J^2 = \pm \mathbb{I}$; 2) $J D = \pm D J$; 3) $J \Gamma = \pm \Gamma J$ (with choice of signs dictated by the KO -dimension of the spectral triple), is said to be real. A real even spectral triple defines a gauge theory, with the gauge fields arising like the inner fluctuations of the Dirac operator

$$D_A = D + A + J A J \quad (2.2)$$

where A is the one form connection given by the commutator of the Dirac operator D and the elements of the algebra, $A = \sum_i a_i [D, b_i]$; the Dirac operator is the product of a continuous part representing space-time, times an internal part of finite dimensional matrices:

$$D = \not{\partial}_\omega \otimes \mathbb{I}_F + \gamma^5 \otimes D_F \quad (2.3)$$

where $\not{\partial}_\omega \equiv \gamma^\mu (\partial_\mu + \omega_\mu)$ and

$$D_F = \begin{pmatrix} 0 & \mathcal{M} & \mathcal{M}_R & 0 \\ \mathcal{M}^\dagger & 0 & 0 & 0 \\ \mathcal{M}_R^\dagger & 0 & 0 & \mathcal{M}^* \\ 0 & 0 & \mathcal{M}^T & 0 \end{pmatrix},$$

$$\text{with } \mathcal{M} = \begin{pmatrix} M_l & 0 \\ 0 & M_q \end{pmatrix}, \quad \mathcal{M}_R = \begin{pmatrix} M_R & 0 \\ 0 & 0 \end{pmatrix}. \quad (2.4)$$

The matrices \mathcal{M} and \mathcal{M}_R , via M_l , M_q , and M_R , contain respectively Dirac and Majorana masses, or better Yukawa couplings of leptons, quark and Majorana neutrinos.

From the Dirac operator D_A , we can deduce the full bosonic action of high energy physics coupled to gravity [7, Sect. 4.1] through the regularization of its eigenvalues,

$$S_B[A] \equiv \text{Tr} f\left(\frac{D_A^2}{\Lambda^2}\right) \quad (2.5)$$

where f is a smooth cut-off function and Λ is the cut-off scale of the order of the unification scale. The parameter Λ is used to obtain an asymptotic series for the spectral action via the heath kernel expansion; the physically relevant terms appear with a non-negative power of Λ as coefficient. One could show that this bosonic action is derivable from its fermionic counterpart via the renormalization flow in presence of anomalies [16–18]. The fermionic action is given by

$$S_F = \overline{J\psi}(D + A + J A J)\psi. \quad (2.6)$$

Let us see the form of the action starting from the formula for a second-order elliptic differential operator D_A^2 of the form

$$D_A^2 = -(g^{\mu\nu} \partial_\mu \partial_\nu + K^\mu \partial_\mu + L). \quad (2.7)$$

This operator can be written using a connection ∇_μ so that

$$D_A^2 = -(g^{\mu\nu} \nabla_\mu \nabla_\nu + E). \quad (2.8)$$

Explicitly, $\nabla_\mu = \nabla_\mu^{[R]} + \omega_\mu$ contains both Riemann $\nabla_\mu^{[R]}$ and “gauge” ω parts, with

$$\omega_\mu = \frac{1}{2} g_{\mu\nu} (K^\nu + g^{\rho\sigma} \Gamma_{\rho\sigma}^\nu). \quad (2.9)$$

Using this ω_μ and L , we find E and compute the curvature $\Omega_{\mu\nu}$ of ∇ :

$$\begin{aligned} E &\equiv L - g^{\mu\nu} \partial_\nu (\omega_\mu) - g^{\mu\nu} \omega_\mu \omega_\nu + g^{\mu\nu} \omega_\rho \Gamma_{\mu\nu}^\rho; \\ \Omega_{\mu\nu} &\equiv \partial_\mu (\omega_\nu) - \partial_\nu (\omega_\mu) - [\omega_\mu, \omega_\nu]. \end{aligned} \quad (2.10)$$

The spectral action has a heath kernel expansion in a power series in terms of Λ^{-1} as

$$\begin{aligned} \text{Tr} f\left(\frac{D_A^2}{\Lambda^2}\right) &= 2\Lambda^4 f_0 a_0(D_A^2) + 2\Lambda^2 f_2 a_2(D_A^2) + f_4 a_4(D_A^2) \\ &\quad + O(\Lambda^{-2}), \end{aligned} \quad (2.11)$$

where the f_k are momenta of the function f ,

$$f_0 = \int_0^\infty u f(u) du, \quad f_2 = \int_0^\infty f(u) du, \quad f_{2n+4} = (-)^n \partial_u^n f(u) \quad (2.12)$$

and the coefficients $a_n(x, P)$ are called the Seeley–DeWitt coefficients [19,20]. They are equal to zero for n odd and the first three even coefficients are given by

$$\begin{aligned} a_0(x, P) &= (4\pi)^{-m/2} \text{Tr}(\mathbb{I}), \\ a_2(x, P) &= (4\pi)^{-m/2} \text{Tr}(-R/6 + E), \\ a_4(x, P) &= (4\pi)^{-m/2} \text{Tr}(-12R_{;\mu}^\mu + 5R^2 - 12R_{\mu\nu} R^{\mu\nu} - 60RE \\ &\quad + 180E^2 + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + 60E_{;\mu}^\mu + 30\Omega_{\mu\nu} \Omega^{\mu\nu}). \end{aligned} \quad (2.13)$$

3. Higgs-singlet scalar potential and gravity

By inserting the relations for the Seeley–DeWitt coefficients (2.13) into (2.11) we obtain the standard model action plus a new singlet scalar field coupled to gravity [21, Eq. (5.49)]:

$$\begin{aligned} S_B &= \frac{24}{\pi^2} f_4 \Lambda^4 \int d^4 x \sqrt{g} - \frac{2}{\pi^2} f_2 \Lambda^2 \\ &\quad \times \int d^4 x \sqrt{g} \left[R + \frac{1}{2} a \bar{H} H + \frac{1}{4} c \sigma^2 \right] \\ &\quad + \frac{1}{2\pi^2} f_0 \int d^4 x \sqrt{g} \left[\frac{1}{30} (-18C_{\mu\nu\rho\sigma}^2 + 11R^* R^*) \right. \\ &\quad + \frac{5}{3} g_1^2 B_{\mu\nu}^2 + g_2^2 \mathbf{W}_{\mu\nu}^2 + g_3^2 \mathbf{V}_{\mu\nu}^2 + \frac{1}{6} a R \bar{H} H + b (\bar{H} H)^2 \\ &\quad \left. + a (\nabla_\mu H)^2 + 2e \bar{H} H \sigma^2 + \frac{1}{2} d \sigma^4 + \frac{1}{12} c R \sigma^2 + \frac{1}{2} c (\partial_\mu \sigma)^2 \right] \end{aligned} \quad (3.1)$$

where $B_{\mu\nu}$, $\mathbf{W}_{\mu\nu}$ and $\mathbf{V}_{\mu\nu}$ are respectively the field strength associated with the gauge groups $U(1)$, $SU(2)$ and $SU(3)$; H is identified with the Higgs field and σ is a singlet-scalar field. This field is related to the neutrino Majorana mass that allows to reproduce a seesaw mechanism of type I as described in [7]. Furthermore, this σ field lowers the standard model Higgs mass to its experimental value. The three momenta f_0 , f_2 and f_4 can be used to specify the initial conditions of the gauge couplings, the Newton constant and the cosmological constant. The coefficients a , b , c , d and e are related to the fermionic Yukawa couplings and Majorana mass matrix and will be written in the crude approximation where the Yukawa couplings of the top quark y_{top} and the neutrino (both Majorana y_{ν_R} and Dirac y_ν) are dominant; in addition, we introduce the dimensionless constant ρ defined by the ratio between the Dirac Yukawa couplings $y_\nu = \rho y_{top}$:

$$\begin{aligned} a &= \text{tr}[y_\nu^* y_\nu + y_e^* y_e + 3(y_{top}^* y_{top} + y_d^* y_d)] \simeq (3 + \rho^2) y_{top}^2, \\ b &= \text{tr}[(y_\nu^* y_\nu)^2 + (y_e^* y_e)^2 + 3(y_{top}^* y_{top} + y_d^* y_d)^2] \\ &\simeq (3 + \rho^4) y_{top}^4, \\ c &= \text{tr}[y_{\nu_R}^* y_{\nu_R}] \simeq y_{\nu_R}^2, \\ d &= \text{tr}[(y_{\nu_R}^* y_{\nu_R})^2] \simeq y_{\nu_R}^4, \\ e &= \text{tr}[y_\nu^* y_\nu y_{\nu_R}^* y_{\nu_R}] \simeq \rho^2 y_{top}^2 y_{\nu_R}^2. \end{aligned} \quad (3.2)$$

Furthermore, it is more transparent to work with the rescaled fields

$$H \rightarrow \left(\sqrt{\frac{2}{3 + \rho^2}} g \right) \frac{H}{y_{top}}; \quad \sigma \rightarrow (2g) \frac{\sigma}{y_{\nu_R}} \quad (3.3)$$

(where g is the gauge coupling to the unification scale) so that the spectral action for scalar fields and gravity reduces to

$$\begin{aligned} S_B &= \frac{24}{\pi^2} f_4 \Lambda^4 \int d^4x \sqrt{g} \\ &\quad - \frac{2}{\pi^2} f_2 \Lambda^2 \int d^4x \sqrt{g} [R + g^2 H^2 + g^2 \sigma^2] \\ &\quad + \frac{1}{2\pi^2} f_0 \int d^4x \sqrt{g} \left[\left(\frac{4}{3 + \rho^2} \right) g^4 H^4 + 2(\nabla_\mu H)^2 \right. \\ &\quad + 8g^4 \frac{2\rho^2}{3 + \rho^2} H^2 \sigma^2 + 8g^4 \sigma^4 + 2g^2 (\partial_\mu \sigma)^2 \\ &\quad \left. + \frac{1}{3} g^2 R (H^2 + \sigma^2) \right]. \end{aligned} \quad (3.4)$$

In the action above, we have neglected the additional gravitational term given by the Weyl curvature. This term is subdominant to the Einstein–Hilbert term at the unification scale [22]. It could be shown [7] that the running of this term changes by at most an order of magnitude at lower scales, so we can assume that it remains subdominant and neglect it in the first approximation. Moreover, we are neglecting the quadratic term in R .

By setting the coefficient f_0 to be $\frac{1}{2\pi^2} f_0 = \frac{1}{4g^2}$ one obtains the normalization of the gauge fields kinetic terms so that the Higgs-singlet potential plus gravity reduces to

$$\begin{aligned} V &= \frac{1}{4} (\lambda_H H^4 + \lambda_\sigma \sigma^4 + 2\lambda_{H\sigma} H^2 \sigma^2) - \frac{2g^2}{\pi^2} f_2 \Lambda^2 (H^2 + \sigma^2) \\ &\quad + \frac{1}{12} R (H^2 + \sigma^2) - \frac{2}{\pi^2} f_2 \Lambda^2 R + \frac{24}{\pi^2} f_4 \Lambda^4 \end{aligned} \quad (3.5)$$

where λ_H , λ_σ , $\lambda_{H\sigma}$ are defined in terms of g , that is the value of the three coupling constants at the unification scale,

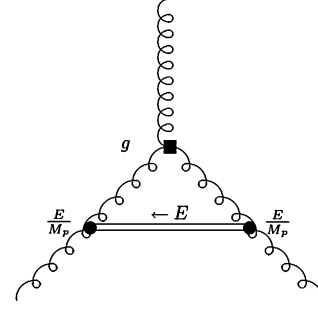


Fig. 1. A typical Feynman diagram at one-loop for a gravitational process contributing to the gauge coupling renormalization. Double lines represent gravitons. Curly lines represent gluons. The three-gluon vertex \blacksquare is proportional to g_i , while the gluon-graviton vertex \bullet is proportional to E/M_P .

$$\lambda_H \equiv \frac{\rho^4 + 3}{(3 + \rho^2)^2} 4g^2; \quad \lambda_{H\sigma} \equiv \frac{2\rho^2}{\rho^2 + 3} 4g^2; \quad \lambda_\sigma \equiv 8g^2. \quad (3.6)$$

The usual strategy, at this point, is to use the spectral action as an effective action at a fixed scale, of the order of the GUT scale $\simeq 10^{17}$ GeV, and to impose the additional relations (3.6) between the independent parameters of the standard model as a boundary condition at that scale. In this Letter we will use a different strategy by shifting the unification scale to the Planck scale M_P . Hence, we want to study the framework in which general relativity is quantized for small fluctuations around a flat space-time, and the Planck scale becomes the real unification scale of all physical interactions. In this extension of the spectral action to higher energy scales, we will include the contribution of gravitons exchange in the running coupling constants. Of course, these contributions will not be significant for low energies and they will be only important near the Planck scale. By using these new RG equations, we can let the standard model parameters run to their value at low scale and test the predictive power of the model: we will obtain a constrain of the free parameters of the theory still compatible with the Higgs and top mass prediction.

4. Gravitational correction to running of Gauge couplings

A possible framework for describing interactions at energies and momenta below the Planck scale is given in [13]. The dynamics for a non-Abelian gauge field coupled to gravity is given by the action,

$$\int d^4x \sqrt{g} \left[\frac{1}{k_{pl}^2} R - \frac{1}{4g^2} \left(\frac{5}{3} g_1^2 B_{\mu\nu}^2 + g_2^2 \mathbf{W}_{\mu\nu}^2 + g_3^2 \mathbf{V}_{\mu\nu}^2 \right) \right] \quad (4.1)$$

where the momentum f_2 is used to specify the initial conditions of the Planck constant, $\frac{2}{\pi^2} f_2 \Lambda^2 \equiv \frac{1}{k_{pl}^2} \equiv M_P^2/16\pi$. The form of the gravitational correction can be determined on a general basis, involving in the one-loop Feynman diagrams of interest a gluon vertex dressed by the exchange of gravitons (see Fig. 1).

Since the gauge boson vertex has strength g_i and gravitons couple to the energy-momentum tensor with a dimensional coupling $\propto 1/M_P$, dimensional analysis implies that the running of couplings in four dimensions will be governed by a Callan–Symanzik β function of the form [13, Eq. 19]

$$\begin{aligned} \beta(g_i, E) &= \frac{b_i}{16\pi^2} g_i^3 + a_g \frac{E^2}{M_P^2} g_i, \\ \text{with } b_i &= \left(\frac{41}{6}, -\frac{19}{6}, -7 \right) \end{aligned} \quad (4.2)$$

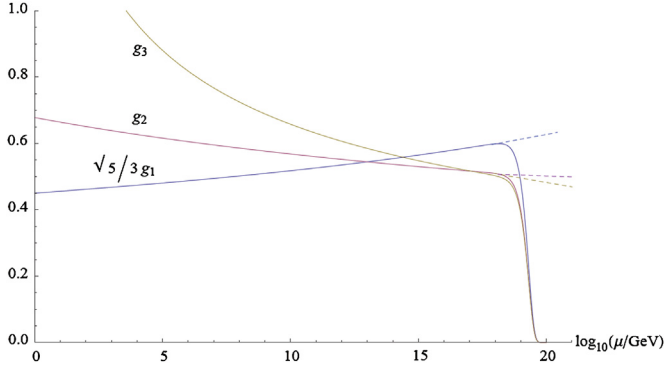


Fig. 2. Including gravity at one-loop, the couplings remain unified near 10^{17} GeV, but evolve rapidly to zero at high E .

where the first term represents the usual standard model contribution, and the second includes the gravitational correction. Initial values of g_i are set with the experimental values at $M_Z \simeq 91$ GeV: $g_1(M_Z) = 0.3575$, $g_2(M_Z) = 0.6514$, $g_3(M_Z) = 1.221$. The numerical value of a_g , also called *anomalous dimension*, is determined by a detailed calculation described in [13] leading to $a_g = -3/\pi$, which we can rewrite $a_g = -\frac{3}{16\pi^2} k_{pl}^2 M_P^2$. The negative sign of this coefficient means that the gravitational correction works in the direction of asymptotic freedom: it forces the couplings to decrease at large energy, as it is shown in Fig. 2. At one-loop order, when gravity is ignored, the three gauge couplings evolve like the inverse logarithm of E (dashed curves); when gravity is included, see the solid lines, the couplings evolve rapidly towards weaker coupling at high E . Of course, its effect only becomes quantitatively important when the energy approaches the Planck scale, and gravitons exchanges are no longer negligible. We finally note that the three gauge coupling constants approximately assume the same value, about zero, from $E \geq 3 \times 10^{19}$ GeV. Near the Planck scale $E \simeq 10^{19}$ GeV the three gauge couplings are not exactly equal: we have $g_1(\Lambda) = 0.372$, $g_3(\Lambda) = 0.386$ and $g_2(\Lambda) = 0.396$.

The unification of the gauge coupling constants, at the Planck scale, has been also considered in several frameworks [24,23] with the request of new fermions, in a different perspective.

5. Renormalization group equations with gravitational corrections

The running of the Higgs mass with the presence of a scalar field has been studied in [11]. However, the RG equations for the matter sector have to be adapted via the addition of the anomalous dimensions of the running parameters, that take into account the contribution of gravity [14],

$$\frac{dx_i}{dt} = \beta_{x_i}^{SM} + \beta_{x_i}^{grav} \quad (5.1)$$

where x_i are the running parameters, $\beta_{x_i}^{SM}$ is the Standard Model beta function for x_i and $\beta_{x_i}^{grav}$ is the gravitational correction. The latter is of the general form,

$$\beta_{x_i}^{grav} = a_{x_i} \frac{E^2}{8\pi M_P^2} x_i(t). \quad (5.2)$$

In our analysis, we use an estimate of the anomalous dimensions as suggested in [14]: a_{x_j} are fixed to 1 for the Yukawa couplings and to 3.1 for the self-interaction couplings of the scalar fields.

For the analysis of the renormalization group flow we shall expand the approach presented in [8,9] with the presence of gravitational contributions. Let M_R be the Majorana mass for the

right-handed tau-neutrino. By the Appelquist–Carazzone decoupling theorem [25], we can distinguish two different energy domains: $E > M_R$ and $E < M_R$.

For high energies $E > M_R$, the renormalization group equations are given by [26, Eq. 15], [27, Eq. B.4] and [28, Eq. B.3], adapted via the addition of the gravitational contributions described above

$$\begin{aligned} \frac{dy_{top}}{dt} &= \frac{y_{top}}{16\pi^2} \left(\frac{9}{2} y_{top}^2 + y_v^2 - \frac{17}{12} g_1^2 - \frac{9}{4} g_2^2 - 8g_3^2 \right) \\ &\quad - a_{y_{top}} \frac{E^2}{8\pi M_P^2} y_{top}, \\ \frac{dy_v}{dt} &= \frac{y_v}{16\pi^2} \left(3y_{top}^2 + \frac{5}{2} y_v^2 - \frac{3}{4} g_1^2 - \frac{9}{4} g_2^2 \right) - a_{y_v} \frac{E^2}{8\pi M_P^2} y_v, \\ \frac{d\lambda_H}{dt} &= \frac{1}{16\pi^2} \left(24\lambda_H^2 - (3g_1^2 + 9g_2^2)\lambda_H + 2\lambda_{H\sigma}^2 \right. \\ &\quad \left. + \frac{6}{16} (g_1^4 + 2g_1^2 g_2^2 + 3g_2^4) \right) + a_{\lambda_H} \frac{E^2}{8\pi M_P^2} \lambda_H, \\ \frac{d\lambda_{H\sigma}}{dt} &= \frac{1}{16\pi^2} \left(6y_{top}^2 + 2y_v^2 - \frac{3}{2} g_1^2 - \frac{9}{2} g_2^2 + 12\lambda_H + 6\lambda_\sigma \right. \\ &\quad \left. + 8\lambda_{H\sigma} \right) \lambda_{H\sigma} + a_{\lambda_{H\sigma}} \frac{E^2}{8\pi M_P^2} \lambda_{H\sigma}, \\ \frac{d\lambda_\sigma}{dt} &= \frac{1}{16\pi^2} (8\lambda_{H\sigma}^2 + 18\lambda_\sigma^2) + a_{\lambda_\sigma} \frac{E^2}{8\pi M_P^2} \lambda_\sigma \end{aligned} \quad (5.3)$$

with $E = E(t) = m_Z e^t$. Below the threshold $E = M_R$, the tau-neutrino Yukawa coupling is replaced by an effective coupling [26, Eq. 14]

$$\kappa = 2 \frac{y_v}{M_R}, \quad (5.4)$$

which gives an effective mass $m_l = \frac{1}{4} \kappa v_0^2$ to the light tau-neutrino. In the range $0 < E < M_R$ the renormalization group equations for λ_σ and $\lambda_{H\sigma}$ are the same, whereas the ones for y_{top} , y_v , and λ_H are replaced by

$$\begin{aligned} \frac{dy_{top}}{dt} &= \frac{1}{16\pi^2} \left(\frac{9}{2} y_{top}^2 - \frac{17}{12} g_1^2 - \frac{9}{4} g_2^2 - 8g_3^2 \right) - \frac{a_y E^2}{8\pi M_P^2} y_{top}, \\ \frac{d\kappa}{dt} &= \frac{1}{16\pi^2} \left(6y_{top}^2 + \frac{1}{36} \lambda_H - 3g_2^2 \right) \kappa - \frac{a_y E^2}{8\pi M_P^2} \kappa, \\ \frac{d\lambda_H}{dt} &= \frac{1}{16\pi^2} \left(24\lambda_H^2 - (3g_1^2 + 9g_2^2)\lambda_H + 2\lambda_{H\sigma}^2 \right. \\ &\quad \left. + \frac{6}{16} (g_1^4 + 2g_1^2 g_2^2 + 3g_2^4) + 12y_{top}^2 \lambda - 3y_{top}^4 \right) \\ &\quad + \frac{a_{\lambda_H} E^2}{8\pi M_P^2} \lambda_H. \end{aligned} \quad (5.5)$$

The numerical solutions to the coupled differential equations (5.3) to (5.5) depend on three input parameters: (1) the unification scale Λ ; (2) the Majorana mass M_R which produces the threshold in the renormalization group flow; (3) the ratio ρ between the Dirac Yukawa couplings of the top quark and neutrino.

The scale Λ , usually taken at the unification $\Lambda_{12} = 10^{13}$ GeV or $\Lambda_{23} = 10^{17}$ GeV i.e. the two extreme points in which $g_1 = g_2$ and $g_2 = g_3$, is now shifted to the Planck scale where, due to the gravitational corrections, the three gauge couplings come together asymptotically free. We will determine the numerical solution from (5.3) to (5.5) for a range of values of ρ , Λ and M_R . The initial

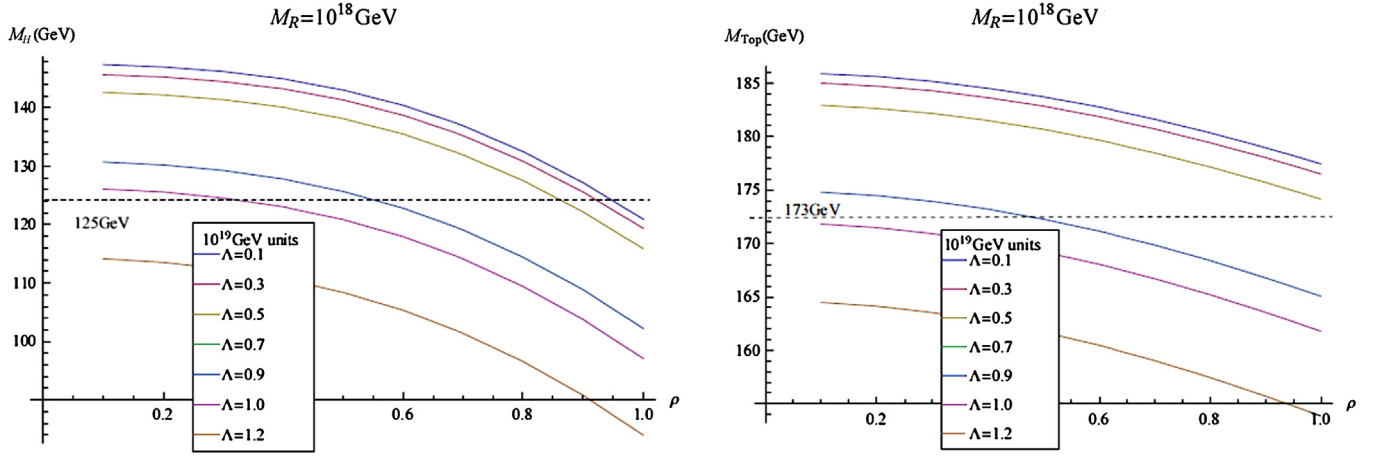


Fig. 3. Higgs and top mass in function of the parameter ρ for seven different values of Λ . We can see that the Higgs mass around 125 GeV and the top mass around 173 GeV constrain Λ not over 1.0×10^{19} GeV.

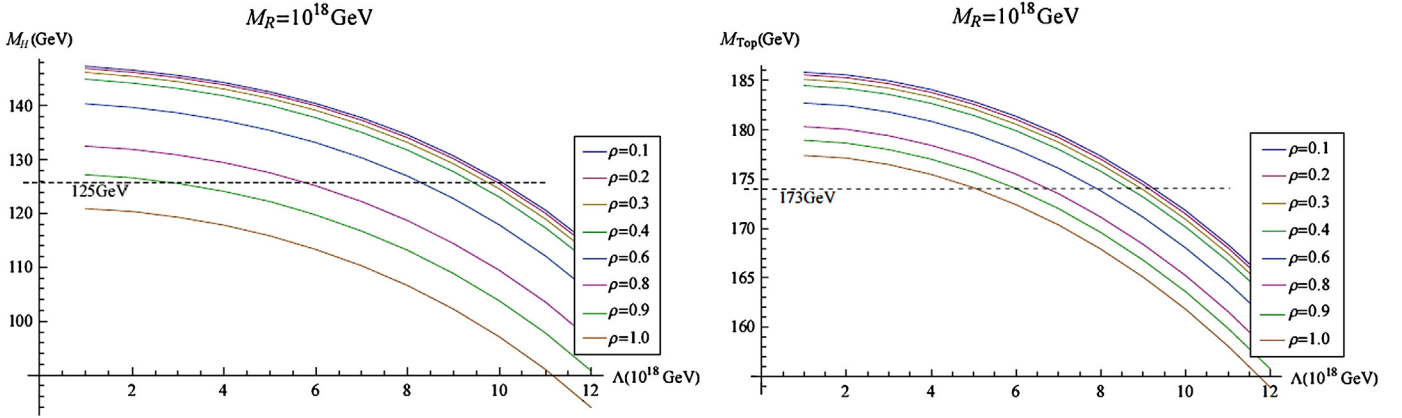


Fig. 4. Higgs and top mass, changing the unification parameter Λ for eight different values of ρ . Also in this case we can see that the Higgs mass around 125 GeV suggests an appropriate choice of ρ not over 1.0 whereas the top mass does not impose any constrain. Moreover both M_H and M_{top} behaviours become ρ -independent for $\rho \leq 0.1$.

conditions of the running parameters at the scale Λ are given by (3.6) plus those for y_{top} and y_ν :

$$y_{top}(\Lambda) = \frac{2}{\sqrt{3+\rho^2}} g_2(\Lambda), \quad y_\nu(\Lambda) = \frac{2\rho}{\sqrt{3+\rho^2}} g_2(\Lambda). \quad (5.6)$$

The effective mass of the light neutrino is determined by the effective coupling κ and we choose to evaluate this mass at the scale M_Z . Moreover, the running mass of the top quark to the ordinary energies is given by

$$M_{top} = \frac{1}{\sqrt{2}} y_{top} v_0 \quad (5.7)$$

where $v_0 \simeq 246$ GeV is the vacuum expectation value of the Higgs field.

For the Higgs mass, we have to use the new relation due to the presence of the new scalar field [11, Eq. 35],

$$M_H(M_H) = v_0 \sqrt{2\lambda_H(M_H) \left(1 - \frac{\lambda_{H\sigma}^2(M_H)}{\lambda_H(M_H)\lambda_\sigma(M_H)} \right)} \quad (5.8)$$

while the scalar-singlet σ mass is proportional to its vacuum expectation value w_0 , near the Planck scale according to us, through [11, Eq. 34], $M_\sigma^2 = 2\lambda_\sigma w_0^2 + 2v_0^2 \lambda_{H\sigma}^2 / \lambda_\sigma$.

The results of the renormalization procedure for the Higgs and top mass in terms of the three parameters ρ , Λ , M_R are shown in Figs. 3 and 4. In Fig. 3 we see the Higgs and top mass values

in terms of ρ for seven different values of Λ and M_R fixed: the Higgs mass around 125 GeV and the top mass around 173 GeV suggest a consistent choice of Λ not over 1.0×10^{19} GeV. In Fig. 4 it is shown the behaviour of the two masses in function of Λ for eight different values of ρ with M_R fixed: also in this case, we can see that the Higgs mass around 125 GeV suggests an appropriate choice of ρ not over 1.0 whereas the top mass does not impose any constrain. Moreover, both M_H and M_{top} behaviours become ρ independent for $\rho \leq 0.1$. Furthermore, it is possible to verify that the parameter M_R is not important for the mass prediction since M_H and M_{top} grow very slowly for its changes. Therefore, in the end, we have a sensible reduction on the choice of the three parameters values.

6. Conclusions

In [12] the new singlet-scalar field σ , responsible for the stability of the Higgs boson, has been derived spontaneously from a high symmetry breaking that occurs at the Planck scale (that means $w_0 \simeq M_P$), mixing space-time spin and gauge degrees of freedom. In this work we have checked the possibility to extend the unification scale up to the Planck scale with the presence of the new scalar field non-minimal coupled to gravity.

We have, then, deduced a restriction of the free parameters of the theory compatible with the Higgs and top mass: in particular, we have to take the parameters $\rho < 1$ and Λ not over 10^{19} GeV. However, this constrain leaves some open problems:

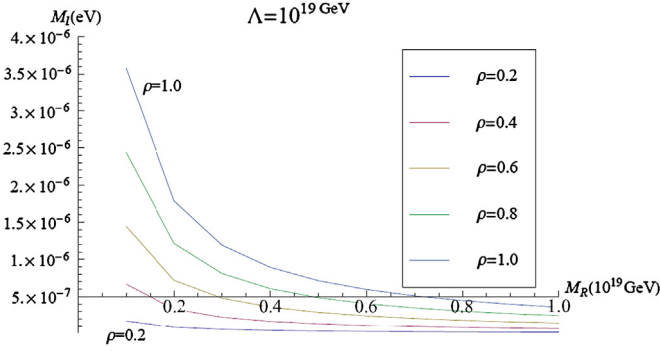


Fig. 5. Neutrino light mass, changing the Majorana right mass value in the range 10^{18} GeV– 10^{19} GeV, for five different values of ρ and the unification scale Λ fixed. We can see that the neutrino mass has a very low value, of the order of μeV . Its value increases for increasing ρ and for decreasing M_R .

for $\Lambda \lesssim 10^{19}$ GeV the three coupling constants are not exactly the same, although very close: e.g. for $\Lambda = 10^{19}$ GeV we have $g_1(\Lambda)^2 = 0.138$, $g_3(\Lambda)^2 = 0.148$ and $g_2(\Lambda)^2 = 0.156$. Actually, we shall take at least $\Lambda \gtrsim 3.0 \times 10^{19}$ GeV to have $g_2(\Lambda)^2 = g_3(\Lambda)^2 = g_1(\Lambda)^2 = 0.003$ and then consistently use the spectral action at the fixed unification scale.

Moreover, we have a neutrino mass problem which now becomes too small since its light mass $m_l = \frac{1}{4}\kappa v_0^2$ is influenced by M_R in the denominator of κ as in (5.4); as shown in Fig. 5 for $M_R \simeq 10^{18}$ GeV the neutrino mass has a very low value of the order of μeV . In order to rise the neutrino mass to few electron-volt, just two actions are possible: (1) increasing the ρ value, but nevertheless there is an upper limit imposed by the Higgs and top mass; (2) lowering the value of the Majorana right mass M_R to 10^{14} GeV. This second possibility seems to indicate that the Majorana right mass (proportional to the σ v.e.v. w_0) responsible for the seesaw mechanism, cannot live at too high energy scales. This observation suggests that we cannot naively identify the scalar field σ of the grand symmetry breaking [12] with the field that gives mass to the Majorana right neutrino; otherwise, there may be some mechanisms that contribute to lower its mass, as in the case of neutrinos. Furthermore, a more punctual analysis is required to investigate the phenomenological consequences of this new and fascinating picture.

Acknowledgement

I would like to thank F. Lizzi for discussions and suggestions.

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